Introduction to Side-Channel Attacks on Elliptic and Hyperelliptic Curves

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Overview

- Groups for DL-systems
- Introduction to side-channel attacks
- Introduction to elliptic and hyperelliptic curves
- Countermeasures against SPA
- Montgomery arithmetic for arbitrary genus 2 curves (Tanja Lange)
- More efficient Montgomery arithmetic for genus 2 curves with element of order 2 (Sylvain Duquesne)

Diffie-Hellman Protocol

Alice

1. secretly generates $a < |\langle g \rangle|$
2. computes $h_1 = a \cdot g$
3. transmits h_1
4. computes $a \cdot h_2$

Bob

1. secretly generates $b < |\langle g \rangle|$
2. computes $h_2 = b \cdot g$
3. transmits h_2
4. computes $b \cdot h_1$

$$a \cdot h_2 = (ab)g = b \cdot h_1$$

Common Key: the group element $k = (ab)g \in G$

Examples (group)

- Diffie, Hellman (1976): $G = \mathbb{F}_p^*$, i.e.

$$G = \{ 1, 2, \ldots, p - 1 \}$$

- $G = \mathbb{F}_q^*$, $q = p^l$
- Koblitz/Miller (1985): $G = E(\mathbb{F}_q)$, group of points on an elliptic curve, i.e.

$$G = \{ (x, y) \in \mathbb{F}_q^2 \, : \, y^2 = x^3 + ax + b \} \cup \{ \infty \}$$

- Koblitz (1989): $G = \mathbf{Cl}(C/\mathbb{F}_q)$, divisor (ideal) class group of a hyperelliptic curve C over $\mathbb{F}_q$
- Using explicit group laws elliptic and hyperelliptic curves are extremely interesting for small devices like smart-cards.

Side-Channels

Attacker can measure

- Time to perform operations,
- Power consumption during operations,
- Electro-magnetic radiation during computation,
- Noise produced during computation.

Obviously, addition is cheaper than multiplication
$\Rightarrow$ needs more clock cycles, different characteristics of power trace.

Attacker might be able to reconstruct sequence of operations (power & EM) or at least learn how many of each kind were performed (timing).

Consequences I

If sequence of operations depends on the secret key and this is directly translated to the observed data, one can reconstruct the key

$\Rightarrow$ Simple Side-Channel Analysis (SSCA) (often SPA$=$ Simple Power Analysis).

(e. g. in binary square-and-multiply one has

S M S S M $\sim$ $(1101)_2 = 13$).

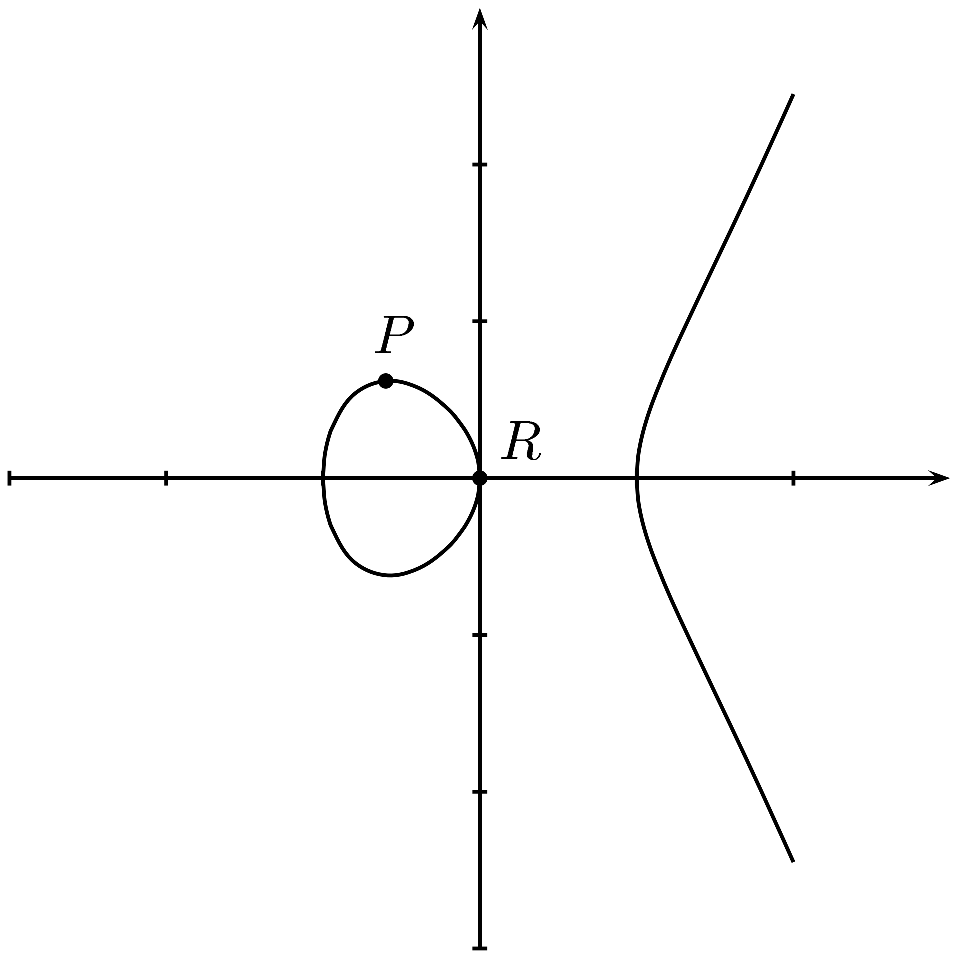
Consequences II

For algorithm shielded against SCA (e. g. by using uniform group operations – topic of these talks) the performance might still depend on the internal representation of the intermediate group elements. Averaging over different inputs reveals intermediate results
$\Rightarrow$ secret key is recovered bit-by-bit from most-significant bit.
$\Rightarrow$ Differential Side-Channel Analysis (DSCA)
(often DPA= Differential
Power Analysis)
not topic of this talk

Elliptic Curve

$$E : y^2 + \underbrace{(a_1 x + a_3)}_{h(x)} y = \underbrace{x^3 + a_2 x^2 + a_4 x + a_6}_{f(x)}, \;\; h, f \in \mathbb{F}_q[x]$$

Group: $E(\mathbb{F}_q) = \{\, (x, y) \in \mathbb{F}_q^2 \; : \; y^2 + h(x)y = f(x) \,\} \cup \{\, P_\infty \,\}$



over $\mathbb{R}$

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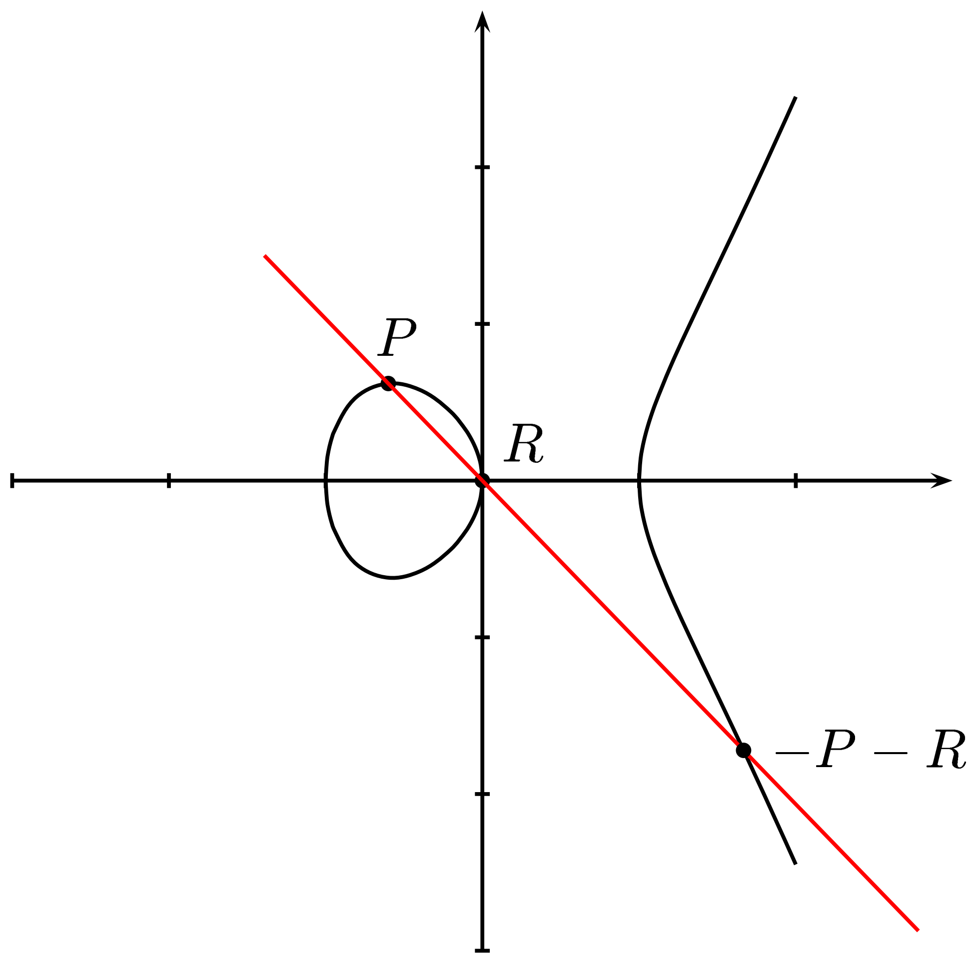

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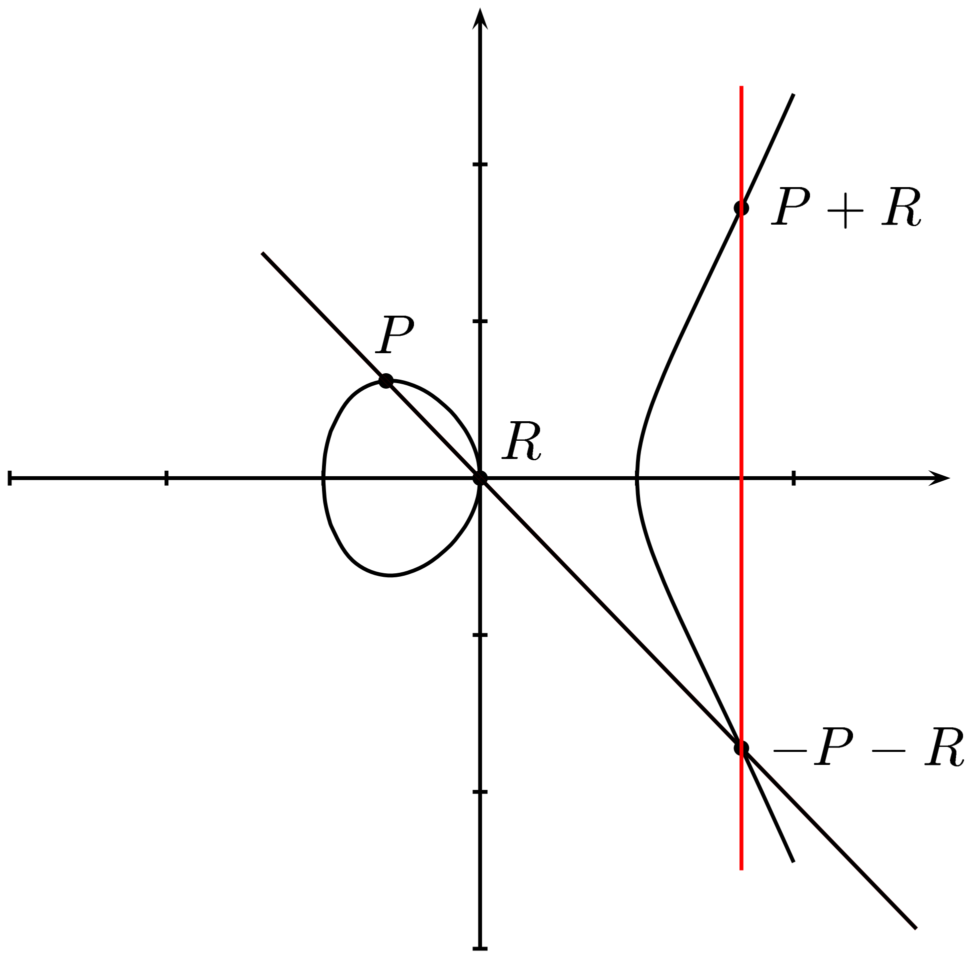

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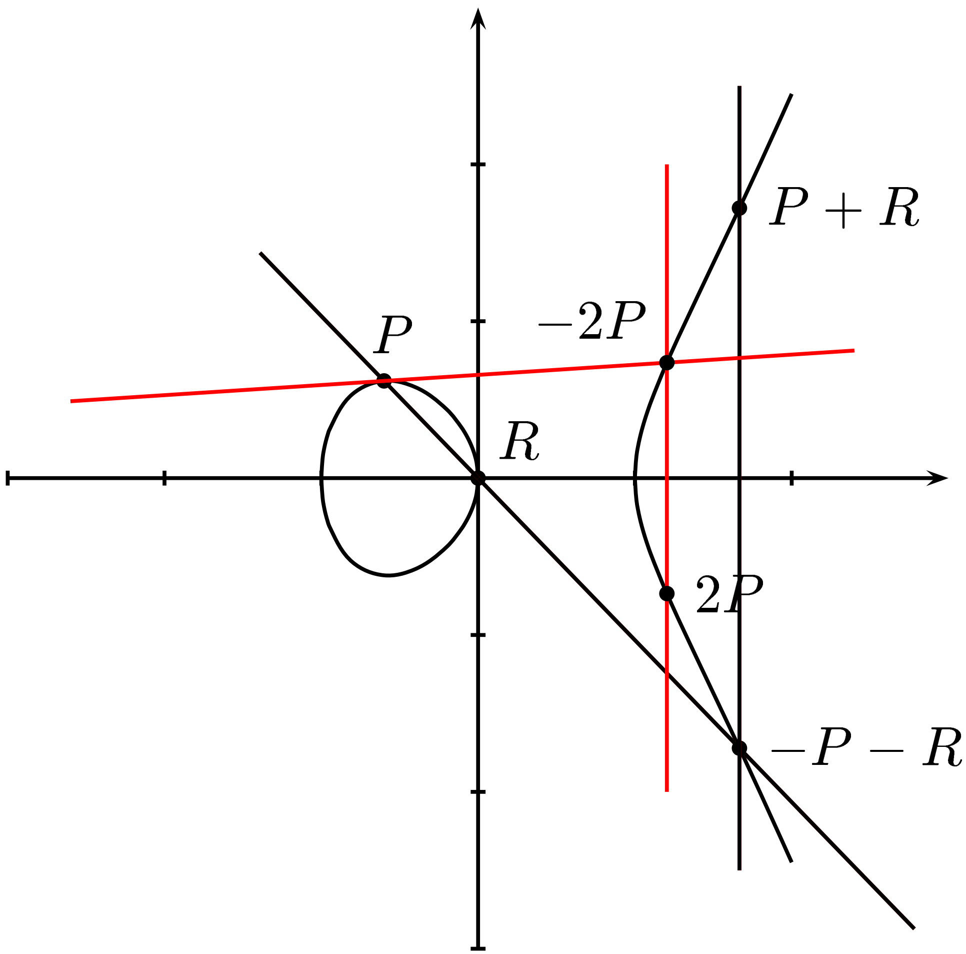

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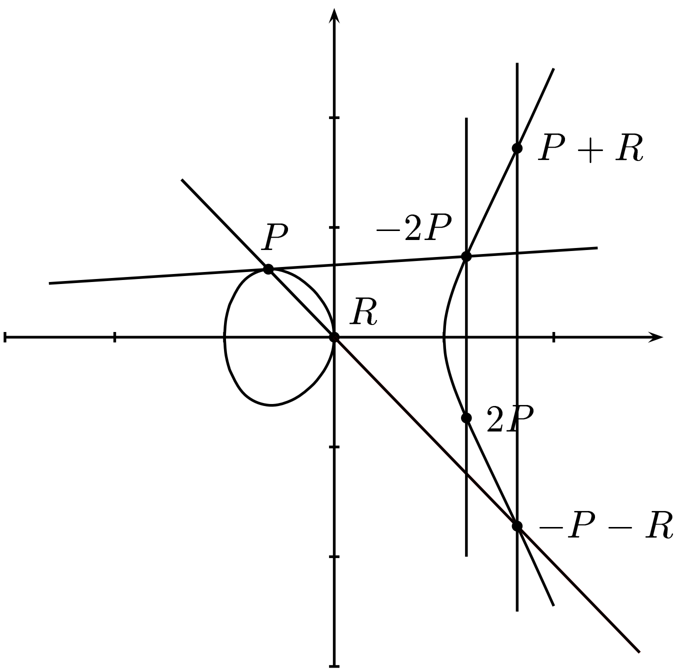
Group Law (q odd)

$$E : y^2 = x^3 + a_4 x + a_6,\ \ a_i \in \mathbb{F}_q$$



$$\begin{aligned}(x_1, y_1) + (x_2, y_2) &= \\ &= (x_3, y_3) = \\ &= (\lambda^2 - x_1 - x_2, \lambda(x_1 - x_3) - y_1),\end{aligned}$$

where

$$\lambda = \begin{cases} (y_2 - y_1)/(x_2 - x_1) & \text{if } x_1 \neq x_2, \\ (3x_1^2 + a_4)/(2y_1) & \text{else.} \end{cases}$$

$\Rightarrow$ Addition and Doubling differ considerably.

1 I, 2M, 1S vs. 1 I, 2M, 2S

Different coordinate systems $y^2 = x^3 + ax + b$

system	points	correspondence
affine ($\mathcal{A}$)	(x, y)	
projective ($\mathcal{P}$)	(X, Y, Z)	$(X/Z, Y/Z)$
Jacobian ($\mathcal{J}$)	(X, Y, Z)	$(X/Z^2, Y/Z^3)$
Chudnovsky Jacobian ($\mathcal{J}^C$)	(X, Y, Z, Z^2, Z^3)	$(X/Z^2, Y/Z^3)$
modified Jacobian ($\mathcal{J}^m$)	(X, Y, Z, aZ^4)	$(X/Z^2, Y/Z^3)$

system	addition			doubling		
affine ($\mathcal{A}$)	2M	1S	1I	2M	2S	1I
projective ($\mathcal{P}$)	12M	2S	–	7M	5S	–
Jacobian ($\mathcal{J}$)	12M	4S	–	4M	6S	–
Chudnovsky Jacobian ($\mathcal{J}^C$)	11M	3S	–	5M	6S	–
modified Jacobian ($\mathcal{J}^m$)	13M	6S	–	4M	4S	–

Scalar Multiplication – Double-and-Add

`IN:` $P \in E(\mathbb{F}_q)$, $n \in \mathbb{Z}, n = \sum_{i=0}^{l} n_i 2^i$
`OUT:` $Q = nP$

1. $Q = P$
2. `for` $i = l - 1$ `down to` 0 `do`
3. $\quad\quad Q = 2Q$
4. $\quad\quad$ `if` $(n_i = 1)$ `then` $Q = Q + P$
5. `output` Q

If ADD $\neq$ DBL one can easily determine n from the sequence of ADD and DBL:

DBL DBL ADD DBL ADD DBL DBL $\Leftrightarrow (101100)_2 = 44$

Countermeasures against SPA

Double-and-always-Add

`IN:` $P \in E(\mathbb{F}_q)$, $n \in \mathbb{Z}, n = \sum_{i=0}^{l} n_i 2^i$
`OUT:` $Q = nP$

1. $Q = P$
2. `for` $i = l - 1$ `down to` 0 `do`
3. $\quad Q = 2Q$
4. $\quad$ `if` $n_i == 1$ `then` $Q = Q + P$
   `else` $R = Q + P$ //dummy operation
5. `output` Q

If an active adversary is allowed, the dummy operations might be detected (fault attacks)

Side-Channel Atomicity

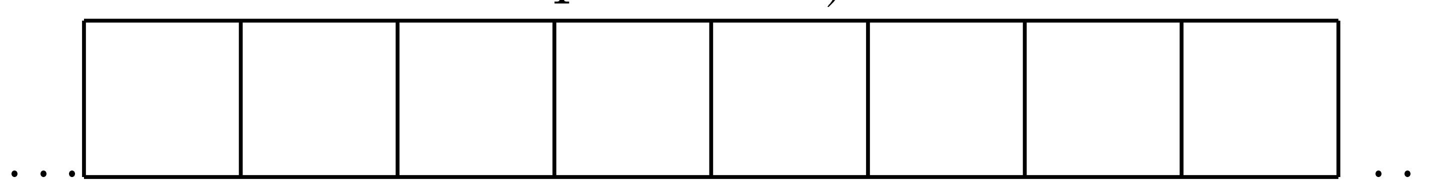
Chevallier-Mames, Ciet, and Joye 2003:
add only cheap dummy operations
$\Rightarrow$ build group operation from identical blocks.

Each block consists of:

1 multiplication, 1 addition, 1 negation, 1 addition;

fill with dummy operations
(only addition and negation $\Rightarrow$ cheap!).

ADD $(\mathcal{A} + \mathcal{P})$ needs 11 blocks
DBL $(2\mathcal{P})$ needs 10 blocks
(same as number of multiplications).



Can be used in parallel/pipelining (Mishra 2004)

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…	ADD$_9$	ADD$_{10}$	ADD$_{11}$	DBL$_1$	DBL$_2$	DBL$_3$	DBL$_4$	DBL$_5$	…

Can be used in parallel/pipelining (Mishra 2004)

Uniform Group Operations

$$\left.\begin{array}{l}\text{Liardet and Smart CHES 2001} \\ \text{Billet and Joye AAECC 2003}\end{array}\right\} \Rightarrow \text{ Jacobi-Model}$$

Joye and Quisquater $\Rightarrow$ Hessian Curve

Proposed formulae still might involve inversions, can be generalized to projective coordinates.

(Some special cases may occur, but with very low probability.)

Montgomery Ladder (Arbitrary Group)

Idea: Make used addition per round.

Consider the intermediate results (i is decreasing).

$Q_i = \sum_{j=i}^{l} n_j 2^{j-i} P$, put $R_i = Q_i + P$, then

$$Q_i = 2Q_{i+1} + n_i P = Q_{i+1} + R_{i+1} + n_i P - P = 2R_{i+1} + n_i P - 2P.$$

This implies

$$(Q_i, R_i) = \begin{cases} (2Q_{i+1}, Q_{i+1} + R_{i+1}) & \text{if } n_i = 0 \\ (Q_{i+1} + R_{i+1}, 2R_{i+1}) & \text{if } n_i = 1 \end{cases}$$

$$13 = (1101)_2 \sim \begin{array}{rcl} (Q_3, R_3) & = & (P, 2P) \\ (Q_2, R_2) & = & (3P, 4P) \\ (Q_1, R_1) & = & (6P, 7P) \\ (Q_0, R_0) & = & (13P, 14P) \end{array}$$

Montgomery Arithmetic for EC

Advantages:

- Needs only x coordinate (not y)
  $\Rightarrow$ lower storage requirement compared to full Montgomery ladder.
- Projective version (no inversion) extremely efficient for special curves (uses x and z) $\Rightarrow$ Montgomery curves (curves were proposed for ECM factoring due to fast group operation).
- Shielded against SPA (Brier/Joye for arbitrary curves), slower than for general curves but faster than full doubling and addition and less storage needed.

Hyperelliptic curve

$$C : y^2 + h(x)y = f(x)$$

$h(x), f(x) \in \mathbb{F}_q[x]$, q prime power,
f monic, $\deg f = 2g + 1, \deg h \leq g$
nonsingular, i. e. not both partial derivatives zero for
$(a, b) \in C/\bar{\mathbb{F}}_q$

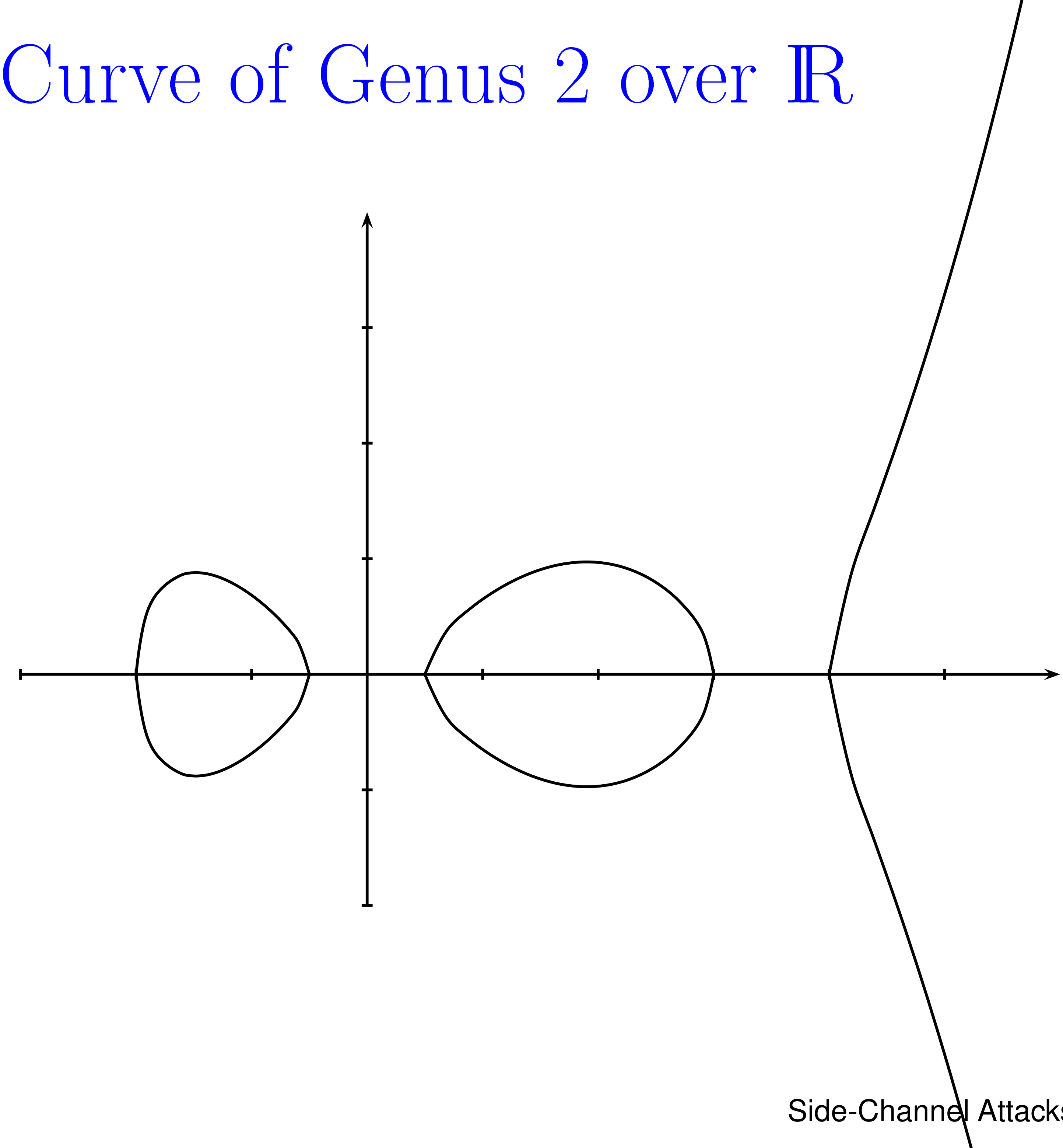
C hyperelliptic curve of genus g

with at least one $\mathbb{F}_q$-rational Weierstraß point
(elliptic curves are seen here as curves of genus 1)

We concentrate on $g = 2$ and odd characteristic > 5

$$C : y^2 = x^5 + f_3x^3 + f_2x^2 + f_1x + f_0.$$

Curve of Genus 2 over $\mathbb{R}$



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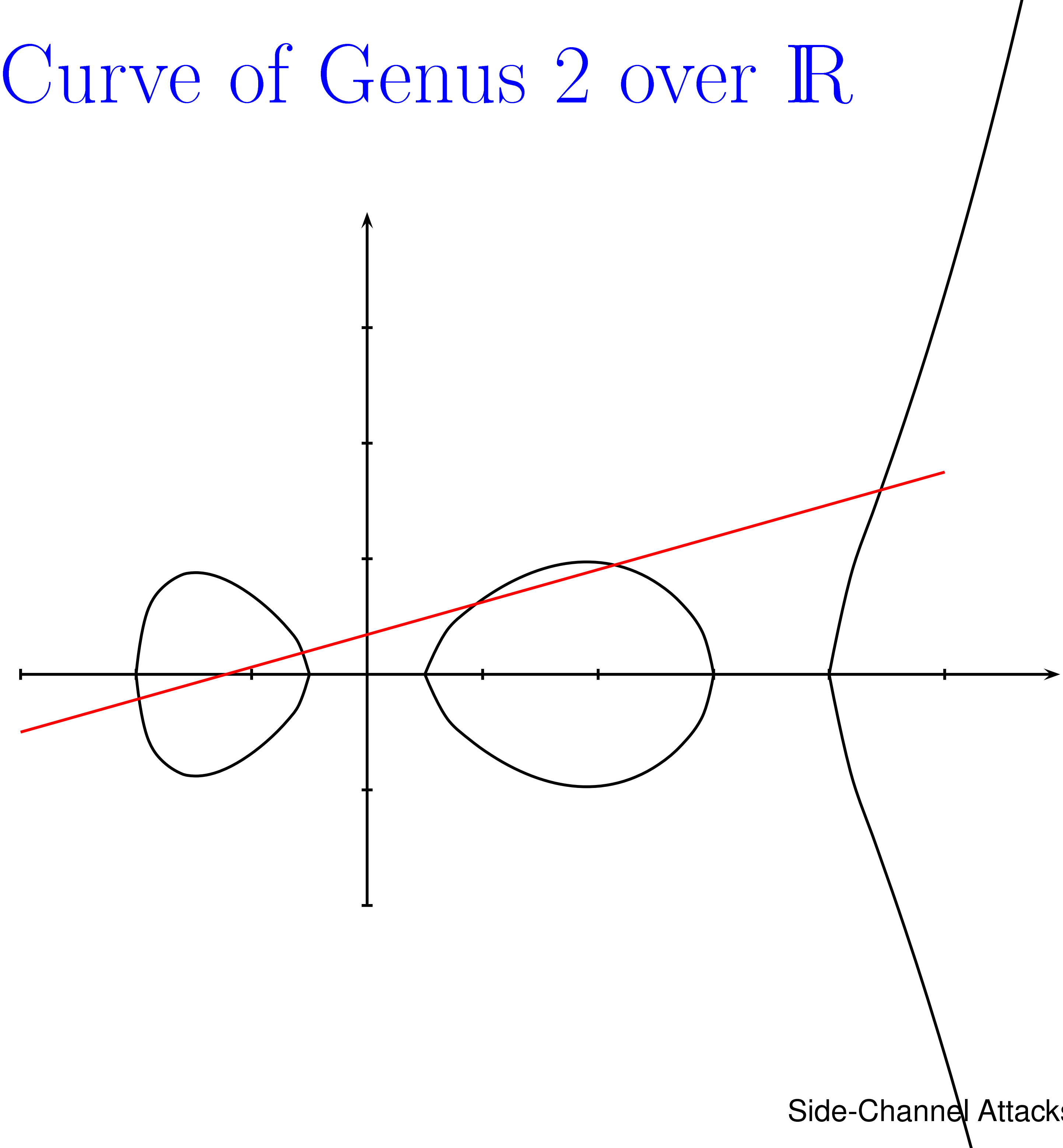

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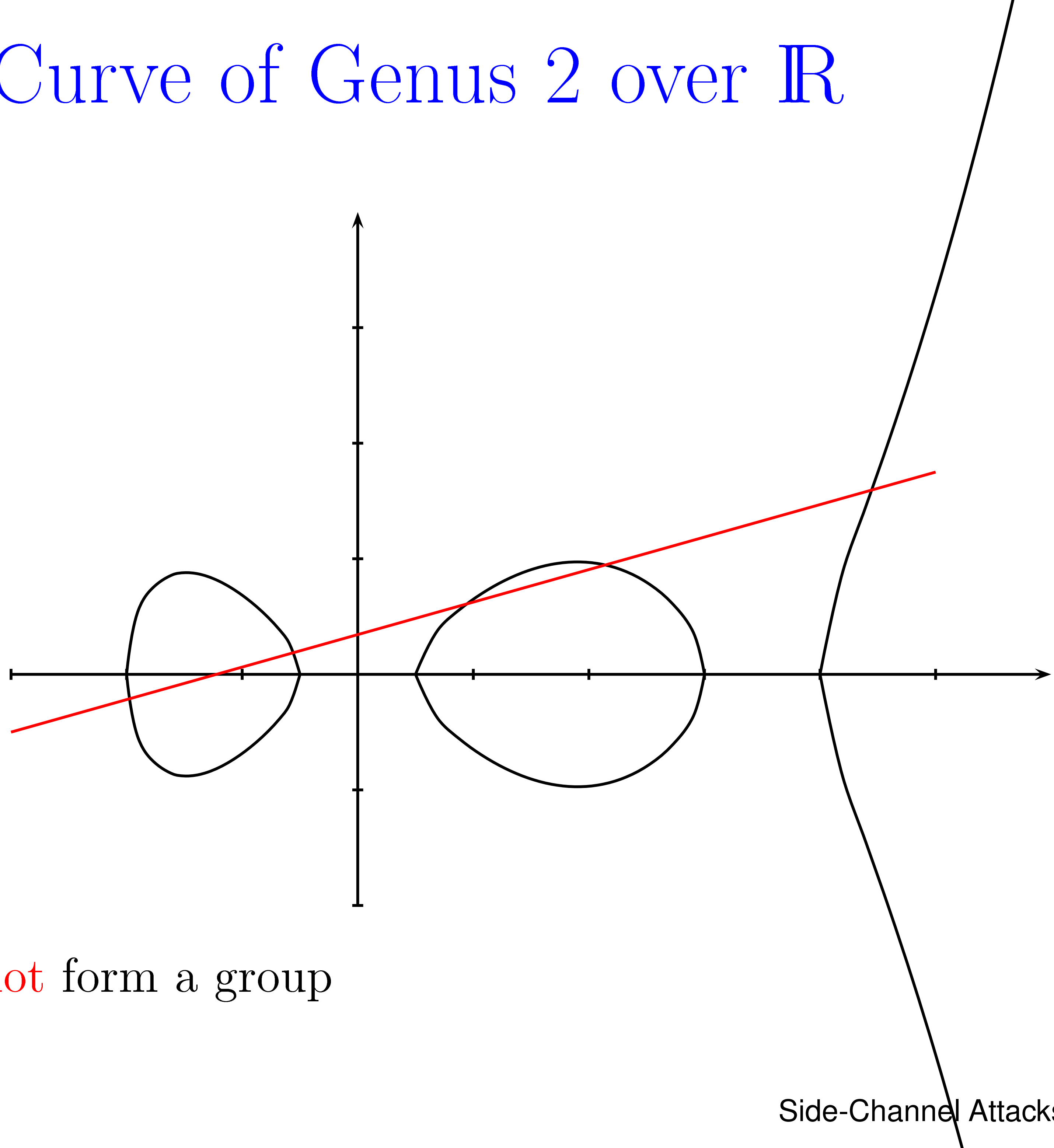


Points do not form a group

Group

Use divisors, i. e. sums of points, of degree zero and reduce modulo principal divisors (divisors belonging to functions)
$\Rightarrow$ divisor class group

$$\mathrm{Pic}^0(C/\mathbb{F}_q) = Div^0(C/\mathbb{F}_q)/P(C/\mathbb{F}_q).$$

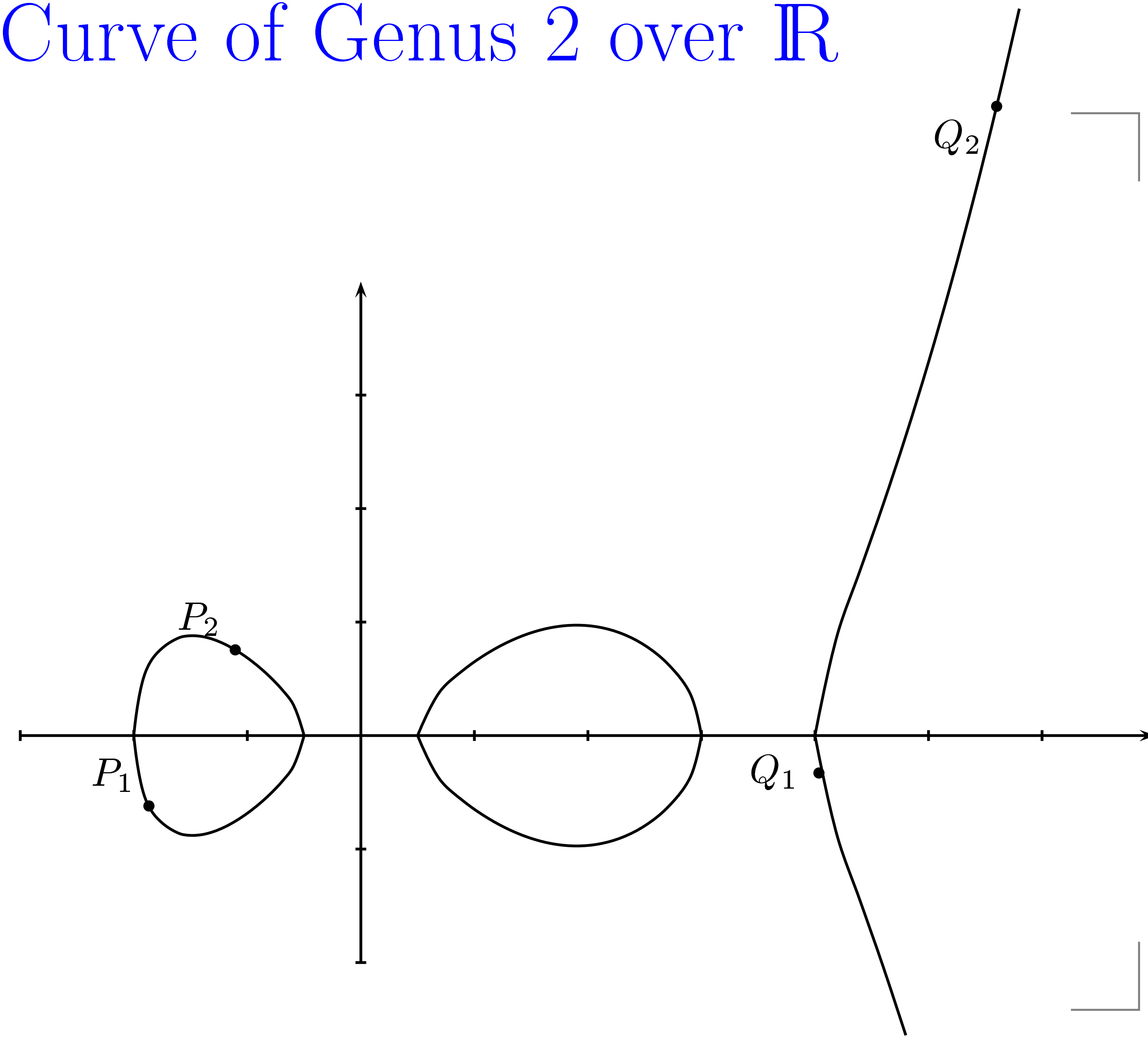
Defined over $\mathbb{F}_q$ means invariant under $\mathrm{Gal}(\overline{\mathbb{F}}_q/\mathbb{F}_q)$.

Each class represented by degree zero divisor with $m \leq g$

$$\sum_{\substack{i=1 \\ P_i \in C \setminus \{P_\infty\}}}^{m} P_i - mP_\infty$$

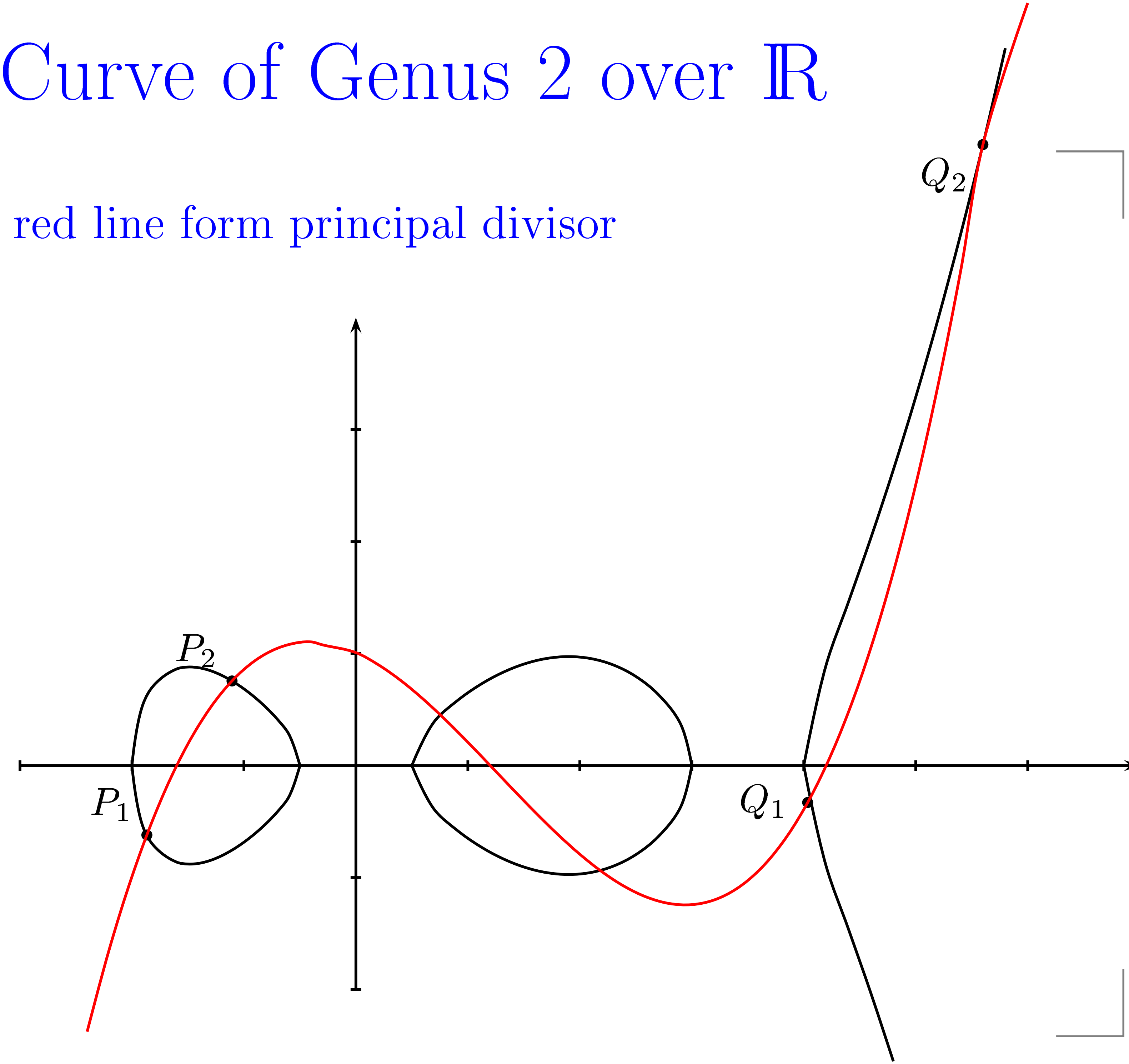
and $P_i \neq \iota(P_j)$ for $i \neq j$.

Curve of Genus 2 over $\mathbb{R}$



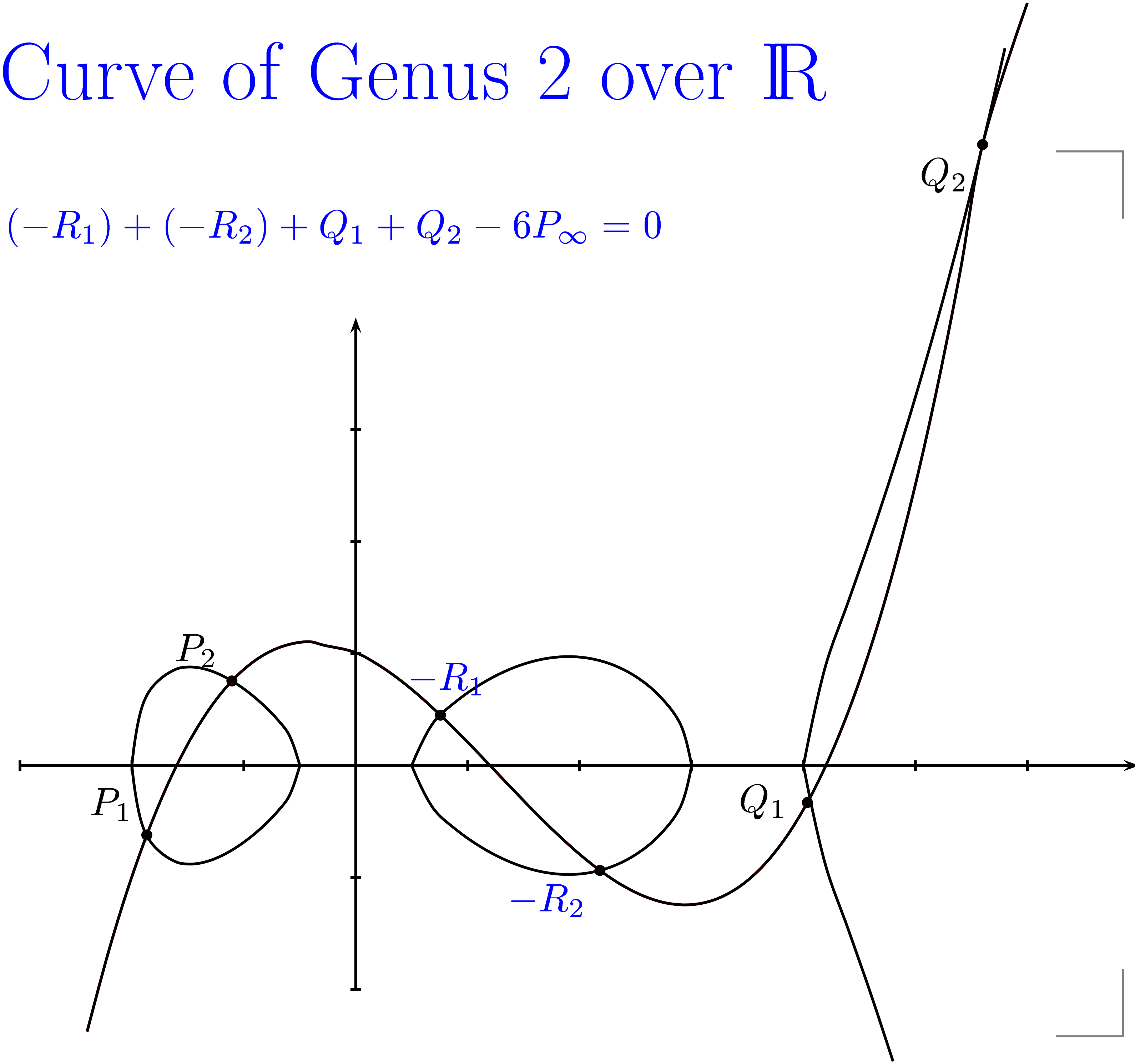
Curve of Genus 2 over $\mathbb{R}$

points on red line form principal divisor



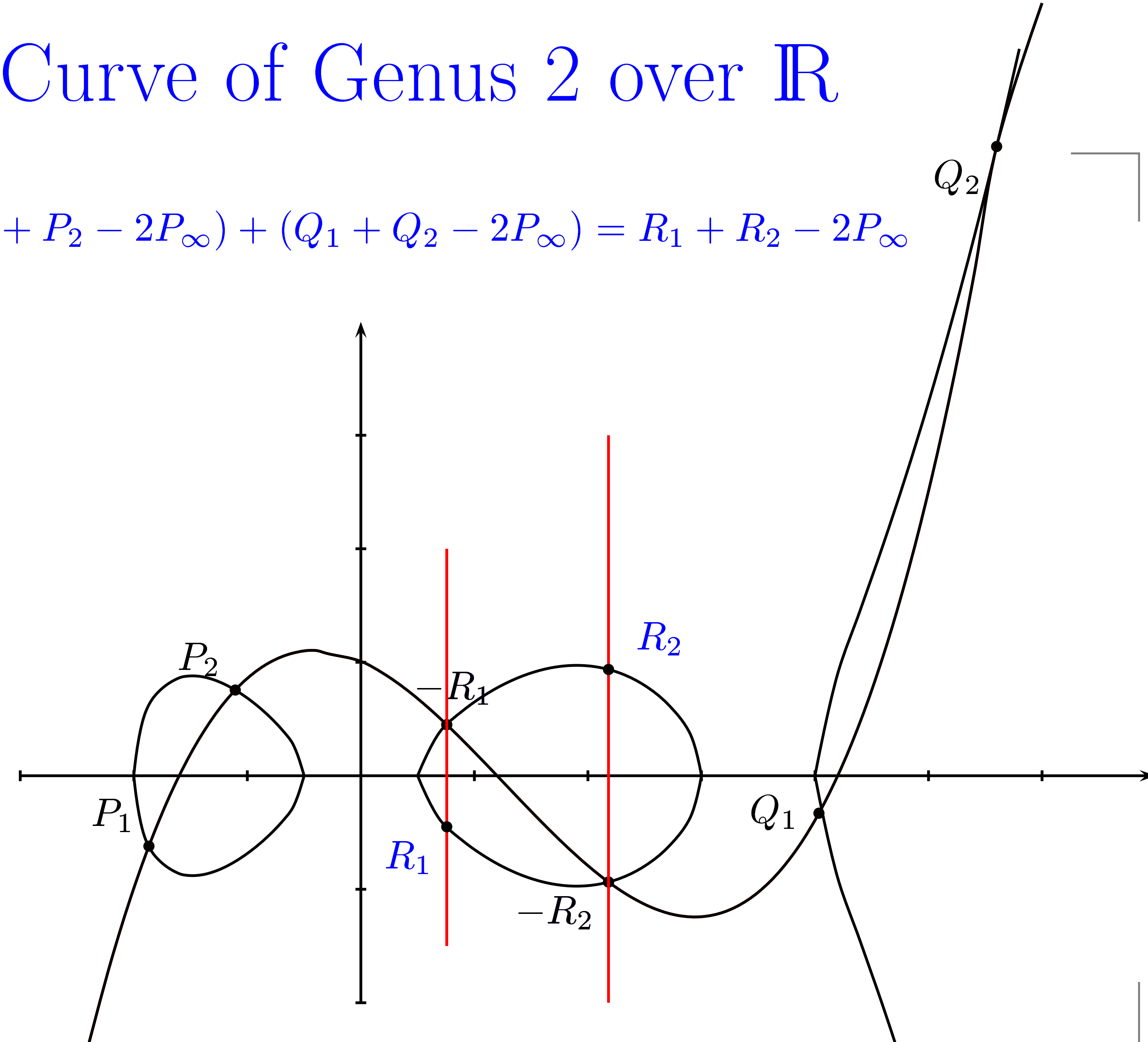
Curve of Genus 2 over $\mathbb{R}$

$$P_1 + P_2 + (-R_1) + (-R_2) + Q_1 + Q_2 - 6P_\infty = 0$$



Curve of Genus 2 over $\mathbb{R}$

$$(P_1 + P_2 - 2P_\infty) + (Q_1 + Q_2 - 2P_\infty) = R_1 + R_2 - 2P_\infty$$



Countermeasures for HECC (SPA)

- Double-and-Always-Add (folklore)
- Atomicity: Lange and Mishra 2004
- Unified group law: still open
- Montgomery arithmetic: Lange, Duquesne (both this ANTS)